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Model Drawing Strategy for Fraction Word Problem Solving of Fourth-Grade Students With Learning Disabilities

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Abstract

This study used a multiple probe across participants design to examine the effects of a model drawing strategy (MDS) intervention package on fraction comparing and ordering word problem–solving performance of three Grade 4 students. MDS is a form of cognitive strategy instruction for teaching word problem solving that includes explicit instruction in drawing bar diagrams to represent problem components. Results suggest the intervention package was effective for improving the fraction word problem solving of students with learning disabilities and that effects were maintained 2 and 4 weeks after intervention. Implications of these findings and indications for future research are discussed.

Keywords

learning disabilities, exceptionalities, learning strategies and direct instruction, instruction, mathematics, elementary, special education, single-subject, research methodology, academic achievement

Fraction knowledge and word problem solving are difficult math topics for many students, including those with learning disabilities (LD; Hecht & Vagi, 2010). Fraction knowledge is an advanced numeracy topic introduced in elementary school and a strong predictor of math achievement in algebra (Booth, Newton, & Twiss-Garrity, 2014; Siegler et al., 2012). The recent standards movement has resulted in states adding algebra courses to graduation requirements, heightening effects of fraction knowledge on students' high-school outcomes (Plunk, Tate, Bierut, & Grucza, 2014). For many students, therefore, learning fractions in elementary school is associated with success in advanced math classes in high school and earning a high-school degree, leading to a higher likelihood of college completion and increased lifetime earnings potential (Gaertner, Kim, DesJardins, & McClarty, 2014).

In addition to fraction knowledge, word problem solving is also an essential competency specified in the Common Core State Standards (CCSS) from kindergarten to Grade 12 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). However, many students with LD experience significant difficulty solving math word problems (e.g., Montague, 2011). This difficulty can be manifested in all phases of Mayer's problem-solving model: problem translation, problem integration, solution planning, and solution execution (Mayer, 1985/2013). Many students with LD struggle to translate problems due to a language-based disability that results in poor reading comprehension and deficits in background knowledge (Vukovic & Siegel, 2010). In the problem-integration phase, students with LD are unlikely to produce a schematic representation that accurately models relations among quantitative elements of a problem (van Garderen, 2006). Moreover, students with LD may experience difficulties in self-regulation for strategic planning (Montague, 2011). Finally, students with LD likely lag behind their peers in overall computation skills needed to use number sentences for problem solving (Kingsdorf & Krawec, 2014). Because students with LD may experience most, if not all, difficulties described above when solving word problems, they need instruction in problem-solving strategies that address each of these areas of difficulty.

Fraction Knowledge of Students With LD

Different from whole-number numeracy that is based in counting discrete items and is additive, fraction knowledge is an advanced stage of numeracy that involves conceptualizing number pairs in multiplicative relationship to each other (Empson, Junk, Dominguez, & Turner,

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2006). Because multiplicative rational number thinking is foundational to algebraic thinking, fraction number knowledge, more than whole-number knowledge, is related to success in algebra (Booth et al., 2014; Siegler et al., 2012).

Two rational number concepts fundamental to fraction knowledge are the part–whole concept of fraction and the magnitude concept of fraction (Torbeyns, Schneider, Xin, & Siegler, 2015). In the part–whole concept, a fraction is a quantity conceptualized by portioning the whole and enumerating the resulting fractional parts (Stafylidou & Vosniadou, 2004). In the magnitude concept of fraction, students put a quantity represented by a fraction in ordinal relation to both fractions and whole numbers. Studying fraction magnitude, students learn fractions can express quantities less than, equal to, or greater than a whole unit. Students also learn equivalent fractions express the same quantity with the same magnitude.

Students with LD demonstrate difficulty learning both the part–whole concept of fraction and the magnitude concept of fraction (Hunt & Empson, 2014). Part–whole understanding involves partitioning and counting requiring sequential processing, whereas fractions magnitude understanding involves thinking of two numbers in a multiplicative relation requiring simultaneous processing (Pantziara & Philippou, 2012). Students with LD may have limitations in simultaneous processing partly due to limited working memory capacity (Jordan et al., 2013).

Students who struggle with fraction concepts also often exhibit whole-number bias by interpreting denominators as they would whole-number quantities (Ni & Zhou, 2005). Whole-number bias affects students with and without LD and results in an inability to flexibly compare fraction magnitudes (Gabriel, Coché, Szucs, Carette, & Rey, 2013). Students with whole-number bias incorrectly apply whole-number concepts to compare fractions with common numerators (e.g., student considers one eighth larger than one fourth because eight is larger than four). Conceptual understanding of the part–whole construct resolves whole-number bias and is evident when the student correctly and flexibly compares fractions with and without common denominators (Mazzocco, Myers, Lewis, Hanich, & Murphy, 2013).

Because students with LD demonstrate persistent whole-number bias as well as difficulty learning part–whole and magnitude fraction concepts (Hunt & Empson, 2014), they need carefully sequenced and targeted instruction to develop conceptual understanding of fraction concepts (Hecht & Vagi, 2010). Such instruction may include using manipulatives, diagrams, or number lines to reduce demands on working memory that inhibit the student's ability to flexibly process relationships among fractions (Misquitta, 2011; Siegler et al., 2010).

Cognitive Strategy Instruction (CSI) for Word Problem Solving

CSI is effective for teaching students with LD math word problem solving (Montague, Applegate, & Marquard, 1993). In CSI, students learn explicit word problem–solving procedures. Intervention studies showed CSI's positive effects with elementary students with LD (Cassel & Reid, 1996; Jitendra et al., 1998; Moran, Swanson, Gerber, & Fung, 2014; Swanson, Lussier, & Orosco, 2013) and secondary students with (LD Montague et al., 1993; Montague, Enders, & Dietz, 2011; Montague, Krawec, Enders, & Dietz, 2014). Evidence suggested CSI supported self-regulation for finding solutions and reduced working memory load by allowing students with LD to break the problem-solving process into a sequence of manageable-sized tasks (Mastropieri, Scruggs, & Shiah, 1991; Zheng, Flynn, & Swanson, 2013).

CSI may include the use of diagrams. For example, Jitendra and colleagues conducted a series of studies using a variant of CSI named schema-based instruction (SBI) that taught students with LD to identify problem types (e.g., change problems and group problems) called schema, and to apply a heuristic for solving them (Jitendra et al., 1998). Specifically, students using SBI first identified semantic relations in word problems that indicated schema type then selected a schema-specific diagram to organize problem information into a solution. SBI was effective for improving word problem solving among students with LD (Jitendra et al., 1998; Jitendra, Hoff, & Beck, 1999; Jitendra, DiPipi, & Perron-Jones, 2002).

With or without using diagrams, most CSI intervention focused on whole-number word problem solving (Cassel & Reid, 1996; Jitendra et al., 1999; Montague et al., 1993; Montague et al., 2014), whereas some studies addressed decimal and proportion word problem solving (Montague et al., 1993). However, to our knowledge, there are no published studies of students with LD using CSI for word problems involving comparing and ordering fractions.

The Model Drawing Strategy (MDS)

One promising approach to teach compare and order fraction word problem solving is the MDS (Dennis, Knight, & Jerman, 2015). MDS is derived from the model method that uses bar diagrams to solve word problems (Kho, 1987; Ng & Lee, 2005). Specifically, both CSI and model methods are included in MDS: It teaches step-by step problem-solving heuristics with a special emphasis on explicitly teaching students how to draw a bar diagram to represent the qualitative relations in word problems.

Although both SBI and MDS teach students to use diagrams, MDS is different from SBI in two major ways. First, in SBI, students use prepared diagrams matching specific problem types, whereas in MDS, students are taught to draw bar diagrams to represent components of any problem scenario. Second, ability to identify problem type is essential in SBI, because subsequent steps rely on selecting the correct diagram for problem solving. MDS does not consider identification of problem type a critical problem-solving step. In MDS, the emphasis is on how to draw a diagram to represent each problem scenario regardless of problem type along with how to use a problem-solving heuristic.

Prior work has shown the model method to be a powerful visual aid for problem solving, because it provides students a global view of an entire problem (Ng & Lee, 2005). The specific feature of drawing a schematic bar diagram in MDS may be especially beneficial for students with LD, because they are less likely than typically achieving students to use schematic diagrams to represent word problems (van Garderen, 2006). Furthermore, because a visual schematic representation of a fraction-related word problem can help clarify the perceived abstract part–whole structure of the notion of fraction (Mazzocco et al., 2013), learning to create schematic diagrams might be especially important for solving fraction-related word problems. Thus, the combination of CSI and model drawing in MDS is intended to improve both students' concept development and their ability to solve word problems.

A recent study showed MDS is a promising approach for teaching fraction and percent word problem solving (Dennis et al., 2015). Dennis et al. (2015) taught three high-school students with LD to use MDS to solve word problems involving adding and subtracting fractions with and without common denominators and one- and two-step percent word problems. Results showed high-school students could learn how to use MDS to solve fraction-related word problems. Although MDS seems to be promising, to our knowledge, no studies exist examining MDS instruction to solve fraction compare and order word problems.

The Present Study

Fraction knowledge is essential, but many students in the United States encounter tremendous difficulty understanding fraction concepts (Siegler et al., 2010). In addition, CCSS call for making sense of quantities and their relationships in problem situations. Embedding fraction-magnitude concepts in word problems requires students to coordinate conceptual and procedural knowledge in real-world scenarios and has been recommended to help students build their understanding of fraction constructs (Gabriel et al., 2013; Hecht & Vagi, 2010).

MDS is a promising approach for teaching word problems involving fraction concepts for students with LD, because the CSI component explicitly teaches them the problem-solving heuristic. In addition, the special model drawing component provides a visual schematic representation of a fraction-related word problem and is aligned to CCSS because it uses a visual fraction model to facilitate understanding of fraction-magnitude concepts. The present study examined the effects of MDS intervention to improve word problem solving involving fraction-magnitude concepts in a sample of fourth graders with LD. This study addresses the following research questions:

> **Research Question 1:** To what extent does the MDS intervention package improve students' performance solving fraction compare and order word problems?
> **Research Question 2:** To what extent do students maintain their word problem–solving performance at 2 and 4 weeks from the end of intervention?
> **Research Question 3:** To what extent do students generalize their conceptual understanding of fraction magnitudes to abstract fraction compare and order problems?

Method

Participants and Setting

Participants included three Grade 4 students; for students' demographic information, see Table 1. To be considered for participation, students achieved a rating of basic or below basic in math at the end of third grade on the Pennsylvania System of School Assessment (PSSA), were identified with LD, and had two math Individualized Education Program (IEP) goals: one for word problem solving and one for calculation. The university's institutional review board and the school's principal granted approval for the research. Parents consented in writing, and students assented verbally to participate in assessment sessions and video recorded individual tutoring lessons. Participants came from the same inclusive classroom setting with push-in special education services.

The study took place in an elementary school in the middle Atlantic region of the United States. The school had 263 students, 64% of whose family incomes qualified them for the federal free lunch program. The first author, a certified special education teacher, taught all intervention lessons in individual tutoring sessions during the school day using either a small room near the main office or a cubicle in the reading specialist's room.

Measures

Dependent measure. Curriculum-based assessment (CBA) probes of fraction word problems aligned with intervention lesson objectives were the dependent measures. The CBA probes were used during the study's baseline, intervention, and maintenance phases. Each CBA probe contained five word problems, each involving one of these fraction skills: (a) compare two fractions with common numerators, (b)

Table 1. Participant Demographic, Cognitive, and Achievement Information.

Participant	Grace	Julia	Wes
Gender	F	F	M
Ethnicity	Hispanic	Hispanic	Hispanic
Age	9.8	9.11	10.3
Grade	4	4	4
Diagnosis	SLD	SLD	SLD/English for Speakers of Other Languages/Speech and Language
Full Scale IQ (Test)	89 (WISC-IV)	102 (WISC-IV)	71 (KABC)
Math Problem Solving (Test)	74 (WIAT-III)	88 (WIAT-III)	81 (KTEA-II)
Numerical Operations (Test)	81 (WIAT-III)	95 (WIAT-III)	77 (KTEA-II)
Reading Comprehension (Test)	89 (WIAT-III)	86 (WIAT-III)	80 (KTEA-II)
End of Grade 3 PSSA Math	Below Basic	Basic	Basic

Note. All reported test scores are standard scores. SLD = specific learning disability; WISC-IV = *Wechsler Intelligence Scale for Children–IV*; KABC = *Kaufman Assessment Battery for Children*; WIAT-III = *Wechsler Individual Achievement Test–III*; KTEA = *Kaufman Test of Educational Achievement–III*; PSSA = *Pennsylvania System of School Assessment*.

compare three fractions with common numerators, (c) compare three fractions using the benchmark-of-one (e.g., when solving the problem: select the largest of three fractions: 5/5, 3/4, 8/6, the student uses 5/5 as the benchmark 1 to determine the relative quantity among the three fractions), (d) order three fractions with common numerators, and (e) order three fractions using a benchmark-of-one.

The researchers generated 18 probes by changing minor elements of the situations and the fractions for each of the problem types. No extraneous information or distractors were included in the word problems. Numerators and denominators of fraction numbers in CBA items were controlled to be consistent with Pennsylvania Department of Education (2014) Mathematic Standards for Grade 4 in that fractions were written with only digits ranging from one to 12. The first author or a graduate student (depending on availability) administered the CBA probes using standardized administration procedures.

Researchers scored each item in the CBA probe using a scoring rubric detailing each of the actions students took to follow MDS steps including each step in drawing and labeling the bars. For example, students earned a point for completing each of the following correctly: listing information for each key problem component; writing information about the question asked in the problem; drawing bar(s) representing the unit; partitioning the bars; shading the bars; and labeling the bars to identify which fraction they represented, providing the correct answer, and presenting the correct answer in a complete sentence. A total of 17 actions were needed to solve each item correctly, with each action given 0 or 1. Therefore, the highest possible point total for each item was 17; the highest possible score for each probe was 85 (5 times with 17 points each). Data presented in Figure 1 are percentage scores calculated by dividing raw scores by 85 and multiplying by 100.

Generalization measure. Generalization test items assessed students' ability to compare and order fraction numbers outside a word-problem scenario indicating transferrable conceptual understanding of fraction magnitude (e.g., students were directed to order the fractions from the smallest to the largest: 5/5, 6/7, 5/4). The tests contained two items of each type: compare fractions with common numerators, compare fractions with benchmark-of-one, order fractions with common numerators, and order fractions with benchmark-of-one. For compare items, students selected the largest of three fraction numbers, and, for order fraction items, they listed three fraction numbers from the smallest to the largest. Items were controlled with all numerators and denominators between one and 12. The researchers scored generalization measures as percentage of items correct, with each item scored either as correct or incorrect.

Social validity measure. Researchers measured intervention acceptability using an adapted version of the *Children's Intervention Rating Profile* (CIRP; Witt & Elliott, 1985). CIRP is a seven-item questionnaire asking students to rate an intervention's usefulness, user friendliness, and disadvantages of participation using a 6-point Likert-type scale from 1 (*do not agree*) to 6 (*agree*). Final score calculations included reversed ratings for items with negative wording. CIRP has internal consistency reliability ranging from .75 to .89.

Treatment fidelity. Researchers recorded intervention lessons for fidelity, but recordings were not available for all lessons because of technical difficulties. There were four lesson videos for Grace, four lesson videos for Julia, and two lesson videos for Wes for a total of 10 total recorded lessons. Graduate students demonstrated the ability in training to use a 12-point fidelity checklist to rate key teacher behaviors during intervention. They were trained to rate the fidelity of implementation in the following areas: models skills expected in the lesson, implements error correction when needed, and allows student to work independently.

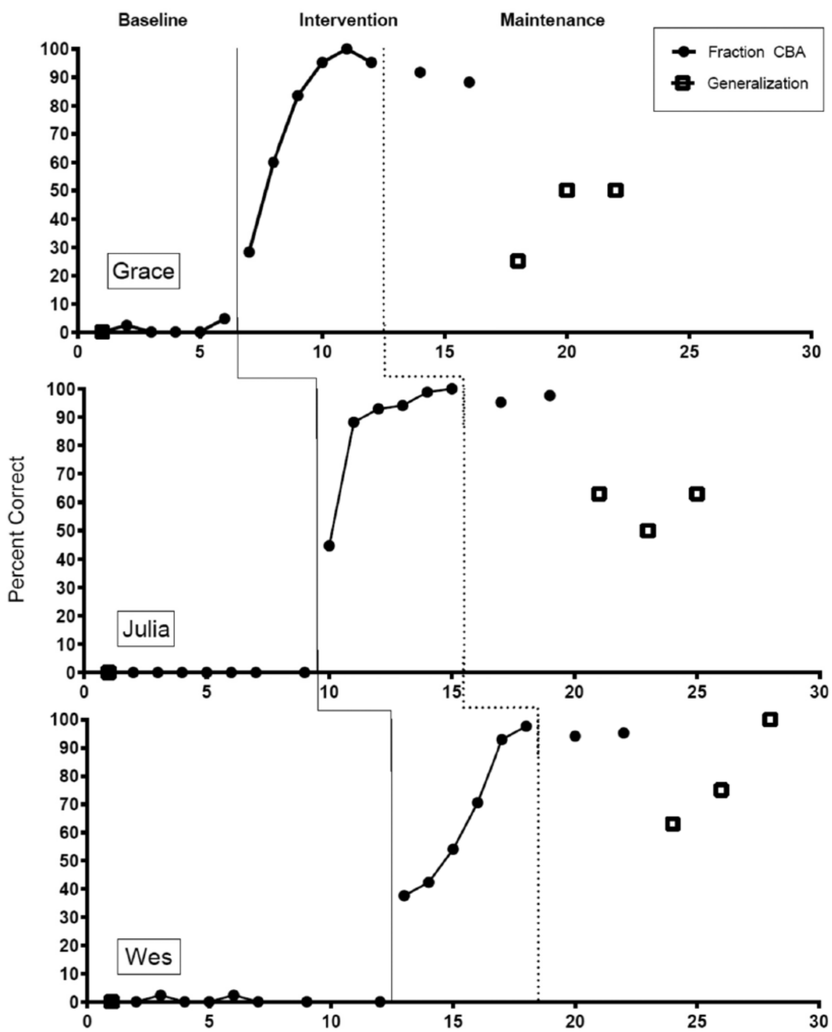

Figure 1. Students' performance for word problem solving.
Note. CBA = curriculum-based assessment.

All videos were scored for fidelity. For interobserver agreement (IOA), a second observer selected two videos for each participant at random and rated the lessons independently. IOA was calculated using the following formula: number of agreements/number of agreements + number of disagreements × 100 (see Table 2 for IOA per participant; the overall implementation fidelity rating for intervention lessons was 98% [range = 92%–100%]; overall fidelity IOA was 100%).

Design

The study used a multiple probe across participants design (Gast, Lloyd, & Ledford, 2014) to determine the effect of intervention. Data collection followed procedures used for multiple baseline single-participant studies. The intervention was implemented in staggered fashion to meet the systematic manipulation criterion (Kratochwill et al., 2013). Repeated-measure data consisted of percentage of correct scores from CBA probes. CBA probes were administered with no time limit, and word problems were read aloud to control for variation in students' reading skills. For testing, students received an MDS steps reference sheet and a stencil for drawing bars. Intervention lessons and probe administrations were conducted in one-to-one sessions. Tutoring pullouts were limited to every other school day. Probe administrations and lessons took place in alternating sessions; intervention probes were given in the next pullout session following each intervention lesson. Tau-*U* index

Table 2. Mean Scores and Standard Deviations on All Measures by Participant.

	Grace		Julia		Wes	
Category	*M*	*SD*	*M*	*SD*	*M*	*SD*
Baseline	1.18	(1.97)	0.00	(0)	0.39	(0.96)
Intervention	77.06	(27.96)	86.47	(20.89)	65.88	(25.51)
Maintenance	90.00	(2.49)	96.47	(1.67)	94.71	(0.83)
Generalization	41.67	(14.43)	58.67	(7.51)	79.33	(18.88)
Tau-*U* index	1.0		1.0		1.0	
CIRP scores	5.71	(0.76)	5.71	(0.76)	6	(0)
CBA interscorer agreement	99.10	99.7	99.80			
Treatment fidelity accuracy	97.23	95.85	100.00			
Treatment fidelity IOA	100.00	97.90	100.00			

Note. Phase data are percentage correct scores on CBA, generalization data are percentage correct scores on generalization probes. CIRP = *Children's Intervention Rating Profile*; CBA = curriculum-based assessment; IOA = interobserver agreement.

calculations estimated intervention effect. Tau *U* was calculated because the statistic takes into account baseline trends that may influence phase contrasts (Parker, Vannest, Davis, & Sauber, 2011).

Procedures

Baseline procedures. Before baseline phase began, students took a pre-intervention generalization test. Baseline data were collected using CBA probes. The initial five data points were collected on consecutive school days. Visual inspection of graphed intervention probe data guided decisions to start participants in intervention according to procedures of multiple baseline study design. Staggered intervention implementation began when the first student's baseline probe performance showed a stable baseline trend. A stable trend was defined as lack of variation in scores so that a line fitting the data in that phase would not show improving performance prior to the start of intervention (Kratochwill et al., 2013). Data collection began in the spring semester after participants' class had completed a fraction unit that included review of fraction concepts and adding and subtracting fractions with like denominators. No math instruction beyond regular classroom math lessons occurred during baseline.

Intervention package. The intervention package included training lessons and MDS lessons. Four training lessons of 30-min duration each preceded MDS intervention lessons. MDS instruction started immediately after the training lessons. During MDS instruction, a probe completion session followed each lesson session on a subsequent day. MDS lessons were video recorded for fidelity scoring but not the training lessons, because we considered the MDS lessons the primary intervention lessons. No additional math instruction beyond regular classroom math lessons occurred during the intervention phase.

Training lessons. The purpose of training lessons was to assure students had prerequisite skills to develop meaningful fraction understandings through word problem solving using MDS. This intervention phase consisted of four 30-min scripted lessons involving instruction in basic fraction concepts. Students took four training lessons after the baseline phase and before they began MDS lessons. No word problems or word problem–solving strategy instruction was included in the training lessons. Each training lesson began with a review of skills taught in the previous lesson. The interventionist first introduced new material using explicit teacher demonstration, then provided guided and independent practice opportunities.

Training Lesson 1 involved introducing fraction vocabulary words including numerator, fraction bar, and denominator. Then, students practiced reading fraction numbers by naming a series of fractions presented on flashcards and writing a series of fractions dictated by the interventionist. In Training Lesson 2, students learned to name fractional parts in fraction strips and fraction bars using total numbers of equal parts (e.g., fourths) and then counting fractional parts to name the numerator (e.g., one fourth, two fourths, three fourths). In Training Lesson 3, students learned to draw a rectangular bar to represent the whole using a cardstock cutout rectangular stencil, to partition the bar to indicate the fractional part represented by the denominator, and to shade in the number of fractional parts indicated by the numerator. Students also learned to represent improper fraction numbers using two rectangular bars drawn adjacent to each other. In the last training lesson, students learned to sort fractions presented on flashcards into three categories: proper fractions, fractions equivalent to one, and improper fractions.

MDS lessons. MDS lessons consisted of six 30-min scripted lessons following immediately after the training lessons. In all intervention lessons, students learned to use

MDS to solve fraction compare and order word problems. The MDS has six steps:

Step 1: Read the problem aloud. The student reads the problem to gain an initial understanding of problem context including givens, resources, and the goal.
Step 2: Decide who and/or what is important. The student identifies and lists relevant information deemed essential for solving the problem.
Step 3: Draw a rectangular bar to show the whole and partition the bar. The student creates a schematic diagram representing the problem. Specifically, the student draws a bar using a stencil and represents relationships between the fraction's parts and the whole by partitioning the bar.
Step 4: Reread each sentence and note known information on the diagram. The student rereads the problem sentence by sentence and marks known information onto the bar diagram.
Step 5: Find the answer to the problem. The student refers back to the schematic diagram and decides the relative magnitude of each fraction.
Step 6: Answer the question. The student writes the complete answer and checks that the answer is reasonable.

The teacher and the student solved three word problems of one of these types in each of the first five intervention lessons: (a) compare two fractions with common numerators, (b) compare three fractions with common numerators, (c) compare three fractions using benchmark-of-one, (d) order three fractions with common numerators, and (e) order fractions using benchmark-of-one. The sixth intervention lesson was a review lesson in which the student used MDS to solve one fraction ordering word problem and one fraction comparing word problem.

All lessons incorporated instructional methods shown to be effective for students with LD, including providing strategy steps for problem solving, teacher modeling, guided and independent practice opportunities, and encouragement for students' verbalizations and summarization of problem content (Montague, 2011). The interventionist started with a *think aloud* demonstrating how to follow MDS steps to solve the first problem from the worksheet. Then, the interventionist used questions and prompts to guide the student through the problem-solving process. Students verbalized during guided practice by reading the problem aloud, explaining the word-problem scenario, naming the whole represented by the bar, stating the relationship between the problem components, and summarizing the information labeling bar diagrams. The interventionist provided corrective feedback when necessary. Finally, the student completed the third word problem independently. Each lesson ended by evaluating the independent practice result and summarizing the big ideas from the lesson.

Students took the package of four training lessons and six intervention lessons over a period of weeks that varied in duration due to snow days and school functions. Grace and Julia spent 10 weeks, and Wes spent 6 weeks in intervention.

Intervention lesson materials. One worksheet with three word problems provided the activities for each lesson. Each problem was printed at the top of a separate page to allow students room for all MDS steps. Students also had an MDS steps poster to refer to during the lesson. A stencil cut from a 3 × 5 index card aided students drawing the rectangular bars.

Maintenance and generalization procedures. Each participant completed maintenance CBA probes at 2 and 4 weeks following their last intervention lesson. These were administered without time limit and with use of a strategy steps poster and a stencil.

Grace completed her first-generalization probe without using MDS steps and earned a score of 35%. The researchers therefore decided to explore whether various levels of prompting would encourage students to use MDS on generalization probe items and whether that would change outcomes on that measure. Thus, participants completed a total of three post-intervention generalization tests with the following administration procedures. In the first test, students received the test with a verbal reminder to use MDS steps to solve the problems but with no other materials provided. The second test was administered with the MDS poster and a stencil, but no reminder to use MDS steps. The final test was administered with all previously listed testing materials and a verbal reminder to use MDS steps to solve the problems.

Social validity. A graduate student who was not the interventionist administered the CIRP questionnaire by reading the questions and recording students' answers (see Table 1 for item means and ranges indicating participants' satisfaction with the MDS instructional method).

Interscorer agreement. Two graduate students scored all CBA probes and generalization probes independently. Interscorer agreement was calculated using the following formula: number of agreements/number of agreements + number of disagreements × 100. Scorers discussed any disagreements and achieved a consensus before data were graphed. Overall agreement for CBA and generalization tests was 99.53% (see Table 2 for interscorer agreement per participant).

Results

Baseline and Intervention

Figure 1 depicts graphed percentage scores for baseline, intervention, maintenance, and generalization phases for the three participants. Baseline performance of all participants was low with little variability (see Table 2 for mean percent scores per participant for each phase). At the beginning of the intervention phase, Grace's data showed a marked change in level, and her performance level trended steadily upward to 100% at the fifth data point. Subsequent participants showed a similar performance pattern starting with a clear change in level from baseline to intervention. Julia's intervention data trended up steeply to above 88% after her second intervention lesson and then accelerated more slowly to 100% for the last two intervention data points. Although Wes's data trend continued upward with unsteady acceleration, he reached 100% performance after the last lesson. In all, visual inspection of the data shows all participants demonstrated marked improvement after intervention was introduced. The overall Tau *U* for the participants was 1.0, indicating a large effect for the intervention package (Parker & Vannest, 2009; see Table 2 for Tau *U* per participant).

Maintenance

Results in Table 2 indicate students maintained the skills they gained from the intervention package at a high performance level for a lengthy period after intervention.

Generalization Test Performance

All participants scored 0% on the generalization measure prior to intervention. After intervention, Grace scored 35% on the first post-intervention generalization measure. She scored 50% on the last two generalization tests when prompted to use MDS steps and provided with materials. Julia performed with 63% and 50% accuracy on the first and second post-intervention generalization probes without using MDS steps. However, given the prompts and materials for the last generalization test, Julia used MDS to answer common numerator and benchmark-of-one items correctly. Wes did not use MDS in any generalization test condition, but his performance improved from 0% before intervention to 63%, 75%, and 100% on tests after intervention.

Social Validity

Results of the CIRP (see Table 2) show high ratings for the intervention package. Students indicated they found MDS intervention a highly satisfactory and useful method to learn to solve word problems involving comparing and ordering fractions.

Discussion

This study examined the effects of MDS intervention package on fourth graders' performance solving fraction compare and order word problems. Results revealed all three students accurately applied MDS problem-solving steps after intervention began. Furthermore, with prompts, students maintained their performance level for 2 and 4 weeks after intervention ended. These findings suggest the intervention package holds promise as an effective approach to improve the fraction compare and order word problem solving of fourth graders with LD.

Before intervention, participants demonstrated whole-number bias through a pattern of answers where fractions with the largest denominators were always selected as the largest (see Figure 2). After intervention, participants were able to use MDS to correctly solve compare and order word problems for fractions with common numerators. Furthermore, they correctly compared proper and improper fractions using the benchmark-of-one strategy (see Figure 3).

The positive results of this study are aligned with previous findings that a step-by-step approach of CSI helps regulate cognitive processes and improve word problem–solving performance of students with LD (Swanson, 2014). Furthermore, findings also suggest that embedding instruction for drawing and using bar diagrams within a CSI may facilitate the problem-solving process for fraction compare and order word problems. The unique MDS feature, drawing and labeling bar diagrams, may have been beneficial to our participants with LD for the following reasons: First, creating diagrams for problem solving may have helped compensate for low working memory. Swanson (2014) and Swanson et al. (2013) have suggested that using schematic diagrams helps students simplify the problem scenario, freeing working memory resources for other task demands. Using bar models, students would have had working memory available to coordinate the conceptual and procedural information for accurate problem solving. Second, students with LD may have difficulty generating schematic diagrams (van Garderen, 2006). Participants may have benefited from explicit instruction in producing schematic bar diagrams. Third, creating bar diagrams may have clarified relevant fraction-magnitude concepts for participants. The bar diagram has been an effective model for teaching fraction and other rational number concepts (Osana & Pitsolantis, 2013; Rittle-Johnson & Koedinger, 2005).

Using a bar diagram labeled with fraction numbers might be especially important for learning to compare and order fractions with common numerators, because it allows students to directly compare the quantities represented in the diagrams. The bar diagram is a flexible schematic representation of a fraction that translates easily to a number line (Keijzer & Terwel, 2003). This opportunity to compare fraction bar models with the fraction number in the visual field is analogous to number line representation and may have helped clarify students' understanding of the inverse relationship of denominators and fraction amounts in unit fractions.

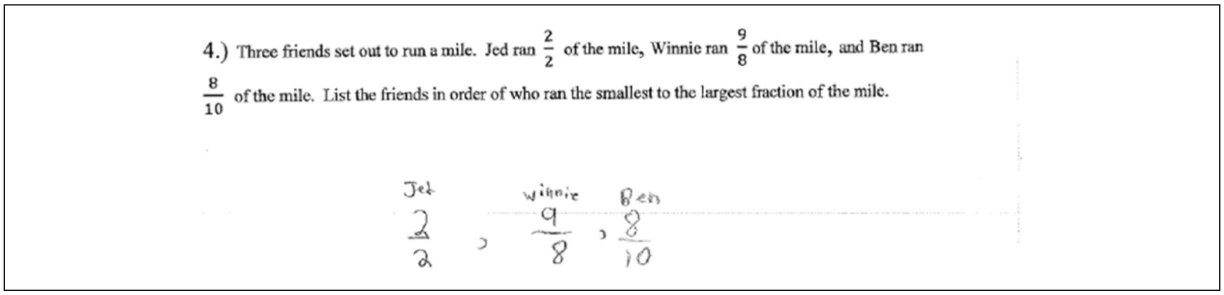

Figure 2. Example of student problem solving before intervention.

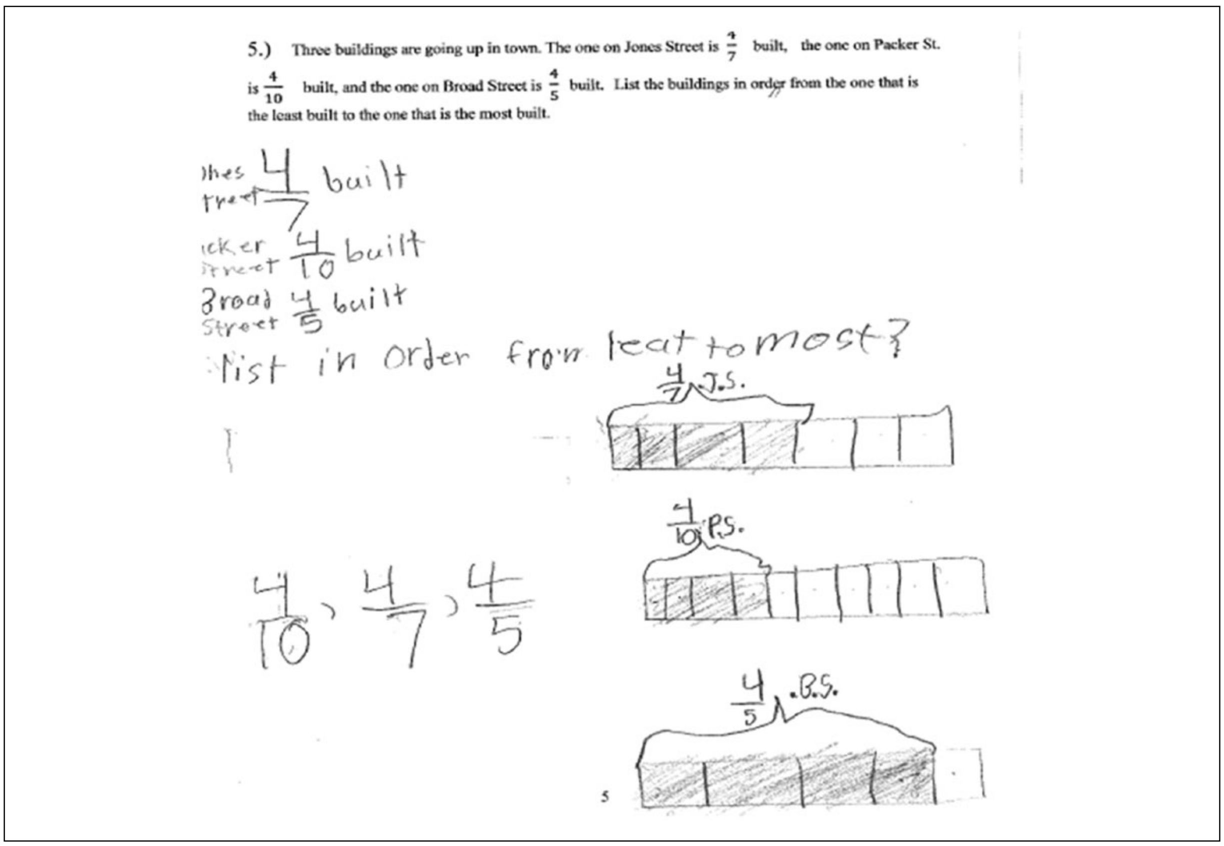

Figure 3. Example of student problem solving after intervention.

Generalization tests assessed changes in students' ability to compare and order symbolic fractions after intervention. The ability to apply fraction concepts in an entirely abstract context post intervention showed transfer from more concrete word-problem scenarios. Accurately and flexibly comparing and ordering fractions both with common denominators and with common numerators would have indicated conceptual mastery. The results showed all participants demonstrated patterns of correct answers on the post-intervention generalization tests. Grace answered all but one of the compare and order items correctly requiring the benchmark-of-one strategy. She got no compare fractions with common numerators items correct, indicating she may have generalized only one fraction concept from the word-problem lessons and did not master the magnitude construct. Grace did not use MDS steps for generalization problems when prompted. Julia correctly answered all items with common numerators and only one benchmark-of-one item until the last generalization test, where, when prompted, she drew bar models and correctly solved all the compare items. She did not use bar models for the ordering problems and got none correct. This indicates Julia did not generalize her understanding of fraction magnitude to abstract problem solving. Wes drew no diagrams in his generalization tests, but scored 100% on the last one indicating he mastered the fraction-magnitude concept.

The variability of students' generalization test performance, especially Julia's and Grace's low post-intervention generalization test performance, was intriguing. Prior research showed students with LD have difficulty conceptualizing fraction magnitudes when required to apply different comparison methods flexibly (Mazzocco et al., 2013). For Grace and Julia, this seemed to be the case. Although both Grace and Julia were able to correctly compare fractions with common numerators when fractions were presented with visual diagrams, they reverted to whole-number bias when fractions were presented as symbols only. To Grace and Julia, it seems the knowledge they obtained at the visual representation level did not directly translate to the abstract symbol level. This suggests that although bar diagrams can help students visualize the relative magnitude of fractions, they need targeted practice to generalize the relationship between the numerator and denominator.

Limitations and Future Research

The present study was the first investigation of MDS for elementary students with LD's solving fraction compare and order problems, and was subject to several limitations. Because the first author delivered the intervention and administered most of the CBA probes, the results are open to potential experimenter bias. In addition, the high fidelity rating for implementation and high level of interscorer agreement were obtained from observers and scorers not trained to a performance criterion. A second limitation was the lack of generalizability of two students (Grace and Julia). Although both students were able to compare and order fractions when they created bar diagrams, they still had difficulty extracting the meaning of fraction at the abstract level. Future research should include fading the visual representation to help students generalize comparing fractions using abstract symbols. Third, the dependent measure was proximal because the CBA probes aligned with the concepts taught in the intervention package. Future studies should include distal measures to further examine the potential effectiveness of MDS. Fourth, the present study has limited applicability, because it used a multiple probe across participants' design with three participants. Although this design meets the single-subject standard, "study must include at least three attempts to demonstrate an intervention effect each at a different point" (Kratochwill et al., 2013, p. 28), the intervention package's positive effects were obtained for only these participants. A larger scale study or replication study using different participants in a similar single-subject design can further demonstrate the intervention's effectiveness. Finally, the present study included 10 lessons, and four of them were training lessons focused on prerequisite skills. The small dosage may have contributed to the lack of generalizability of two participants. Future research should extend intervention duration to solidify students' understanding of lesson concepts. In addition, because both training lessons and MDS lessons took place between baseline and intervention, and no data were taken during training lesson implementation, the participants' positive performance can be attributed to the whole intervention package rather than to the MDS lessons exclusively. To further examine the effectiveness of MDS, future studies should provide training lessons prior to baseline data collection. Another possible approach would be to select participants who possess required skills.

Instructional Implications

The present study showed the intervention package has promise for improving foundational fraction knowledge of elementary-level students with LD. Fourth-grade students with LD learned to use MDS steps to solve fraction compare and order word problems. The unique component of MDS was students drawing and labeling bar diagrams themselves, giving them the opportunity to engage in deeper processing of fraction magnitudes (Booth & Siegler, 2008). In addition, students were prompted to name the whole represented by the rectangular bar, providing verbal mediation in addition to visual stimulation that may have aided conceptual integration by focusing students' attention on the fraction as a meaningful unit. Although future research is needed to further confirm the effectiveness of MDS, these findings suggest the MDS intervention package is a promising approach to help students with LD successfully solve fraction compare and order word problems.

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